

# MODELING URBAN HOUSING MARKET DYNAMICS

“Don’t buy the house, buy the neighbourhood” (Russian proverb)

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Housing market : Designing the model

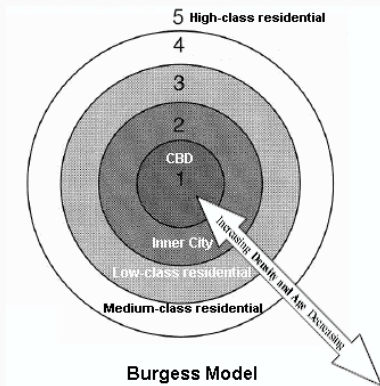
Housing market model : Adaptation to the city of Paris

## CONTEXT

Emergence of **collective phenomena in socio-economic systems** from interactions between elementary entities

⇒ socio-spatial segregation through local **residential dynamics**

- Chicago school (1920)
- Schelling (1978)  
“Micromotives and Macrobehavior”
  - organized discrimination
  - individual incentives



## OUTLINE

- Population distribution in the city investigated **through residential dynamics**
- How individuals heterogeneous in income are spread around the city?
- Assumptions on the **individual preferences**
  - Development of a general model
  - Adaptation to the city of Paris
- Measuring the social diversity

# HOUSING MARKET : DESIGNING THE MODEL

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## Housing market : Designing the model

Housing market model : Adaptation to the city of Paris

## FORMATION OF PRICES ON THE REAL ESTATE MARKET

### Rosen (1974), hedonic prices

- Prices depend both on **intrinsic** variables (number of rooms, size) and on **extrinsic variables** concerning the area and the facilities.
- *Brueckner et al. (1999)* : the relative location of different income social groups depends on the **spatial pattern of amenities in a city**
- The role of the **quality** and density of the **neighborhood**, the reputation of the neighboring schools and the level of security matter : *Ioannides (2003)*, *Figlio and Lucas (2004)*, *Bono et al. (2007)* or *Seo and Simons (2009)*.

⇒ In our model, the formation of prices depends both on an intrinsic attractiveness and a dynamic subjective one which depends on the willingness to pay of the agents.

## SOCIAL PREFERENCES IN RESIDENTIAL DYNAMICS

Models of residential with social preferences (Schelling (1971)) were enriched with an economic component

- Zhang (2004) : utility function as a **trade-off between minimization of the transaction prices and maximization of individual preferences**
- There were no notion of **accessibility** (anyone can buy) or **attractivity of the places** like in Fossett and Senft (2003)
- Bernard and Willer (2007) also introduced the notion of social distances in their model

⇒ **Our model takes into account the social composition (measured by the levels of income) of the neighborhood and is data-driven**



## FRAMEWORK [1]

Location within an urban area is a determinant affecting house prices. The attractiveness of an house should then depend on the location.

- **Monocentric model**

Alonso model [Alonso 1964](Mills, Muth, Von Thünen...)

There exists a link between demand for cultural amenities and income. Amenities tend to be located in the center in European cities [Brueckner, Thisse, and Zenou (1999, EER)]

⇒ **Distance to the center**

$$A^0(X) = A_{max} \exp\left(-\frac{D(X)^2}{R^2}\right) \quad (1)$$

## FRAMEWORK [2]

- **Residential choice** : role of the attractiveness [Alexandre et al 2010]  
Amenities  
Individual preferences (neighborhood) [Schelling 1971]  
  
⇒ **Attractiveness**
- **Urban segregation** : economic dimension  
Economic constraints affect the organization of the individuals [Durlauf 1992, Bennabou 1993, De Bartholome 1990]  
  
⇒ **Economic factor**

## MODEL OVERVIEW

Real estate transactions between heterogeneous agents

- Spatial representation of the city
- Location/Good characterized by an **attractiveness**
- Prices of these goods depend on the attractiveness
- **Agents characterized by their Willingness To Pay** (income) arrive in the city
- These agents choose a place to live (a good) according to its relative attractiveness
- Attractiveness updated according to the last agents distribution in the city
- Some agents are moving out

## RESIDENTIAL DYNAMICS

At each period, there is a finite number of agent, who can be in one of the three following states :

1. buyer
2. seller
3. housed

We assume an infinite “reservoir” of agents outside the city. Agents in the reservoir are heterogeneous in their income—they are indiscernible except for their income category.

## MODEL PARAMETER [1]

- Reserve prices included in [100000, 325000]

$$P_{k,k \in \llbracket 0, K-1 \rrbracket}, k = \text{category} \quad (2)$$

high category  $\equiv$  high income (reserve price)

$P_k$  is the highest price a buyer is willing to pay for goods

- Uniform distribution of the WTP  $P_k = P_0 + \frac{k}{K}inc$  (linear relationship with the category)

## MODEL PARAMETER [2]

- **Attractiveness** : static location-intrinsic part + dynamical part

$$A_k(X, t = 0) = A_0(X) \text{ intrinsic attractiveness} \quad (3)$$

$$\Delta A_k = \underbrace{\epsilon' v_{k \geq}(X, t)}_{\text{New settlers of higher categories}} + \underbrace{\omega \Delta t (A_0(X) - A_k(X, t))}_{\text{Relaxation term}} \quad (4)$$

- $A_0(X)$  can for example be a measure of the **density of amenities**, here we assume that it decreases with the distance to the center
- $A_k(X, t)$  is a measure of the attractiveness seen by an agent in the category  $k$ , it depends on the **composition of the neighborhood**

## MODEL PARAMETER [3]

- **Bottom prices** :the willingness to sell or each  $k$ -agent acting as a seller is determined by his WTP,  $P_k$ , and by the intensity of the demand, through the level of the mean attractiveness of the location :

$$P_l(X, t) = P^0 + (1 - \exp(-\lambda \bar{A}(X, t)))P_k \quad (5)$$

- A transaction between a  $k$ -buyer, with demand price  $P_{k_1}$ , and a  $k$ -seller with offer price  $P_{k_2}$  can be completed if  $P_{k_1} > P_{k_2}$ . If such a transaction occurs, the **transaction price** is assumed to be a linear combination of offer and demand prices

$$P_{tr} = (1 - \beta)P_{k_2} + \beta P_{k_1} \quad (6)$$

where  $\beta$  is a constant coefficient

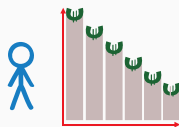
## MODEL PARAMETER [4]

- number of agents arriving on the market
- number of agents becoming sellers



## SCHEMATIC REPRESENTATION OF THE DYNAMICS

Heterogeneous agents



probability to  
choose a location  $x$

$$p(x) = \frac{1 - \exp(-\lambda A_k(X, t))}{\sum_{x \in \Omega} 1 - \exp(-\lambda A_k(X, t))}$$

City with residential locations



different attractiveness  
different prices

newcomers

agents not housed  
not enough assets  
too small WTP

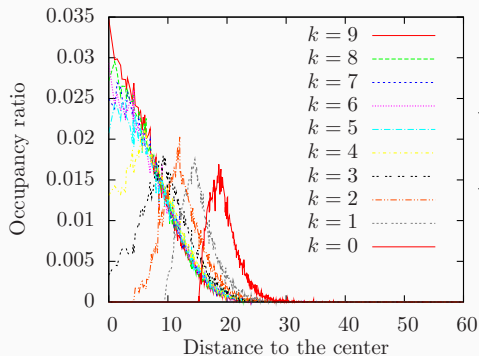


agents housed



The last transactions increase  
the attractiveness of a location seen  
by agent with "less income"

## SIMULATION RESULT : SOCIO-SPATIAL SEGREGATION AND INCOME MIX

Occupancy ratio per WTP versus distance to the center3 zones-segregated : occupied  
by higher WTP-segregated : occupied  
by lower WTP

-social mix

 $K = 10$  levels of revenues

↑ Social mix ?

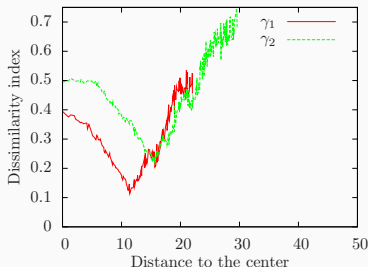
## SOCIAL MIX INDEX

The **segregation** is measured with the following dissimilarity index (Reardon, Firebaugh) :

$$ID(X) = \sum_{k=0}^{K-1} \left| \nu_k(X) - \frac{1}{K} \right|. \quad (7)$$

where  $\nu_k(X)$  is the relative density of  $k$ -agents :

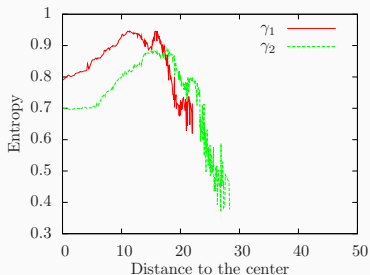
$$\nu_k(X) \equiv \frac{u_k(X)}{\sum_{k=0}^{K-1} u_k(X)}, u_k(X) \equiv \text{occupation number} \quad (8)$$



## SOCIAL MIX INDEX

## Entropy measure

$$H(X) = - \sum_{k=0}^{K-1} \nu_k(X) \log \nu_k(X) \quad (9)$$



## WTP Threshold

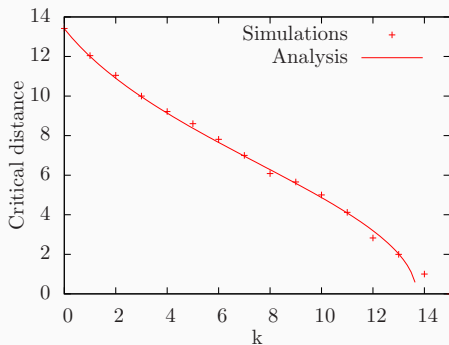
∃ WTP threshold  $P_C^*$  :

- if  $P_k \geq P_C^*$ ,  $k$ -agents are able to **locate anywhere**
- if  $P_k < P_C^*$ ,  $k$ -location depends on a **critical distance** which depends on the agent's WTP.

Threshold results from the dynamics, depending on the subjective attractiveness, which depends on the neighborhood.

⇒ **Two types of agents : above and below the threshold**

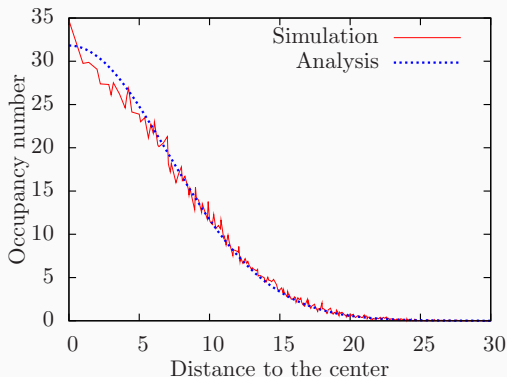
## Evolution of the critical distance with respect to $k$



$$d_c^*(k) = R \sqrt{-\ln\left[\frac{\frac{1}{\lambda A_{max}} \ln \frac{P_k}{P_0}}{1 + \frac{\epsilon \gamma}{\omega k^2} \left[ \frac{(k+1)k}{2} - \frac{(k+1)k}{2} \right] \sum_{X'} A^0(X')}}\right]}$$

## COMPARISON BETWEEN ANALYTICAL AND SIMULATION RESULTS (1)

Agents with Willingness To Pay above the threshold



$$\forall X \in \Omega, u_k^*(X) = \frac{\gamma}{K\alpha} \frac{A^0(X)}{\sum_{X \in \Omega} A_0(X)}$$

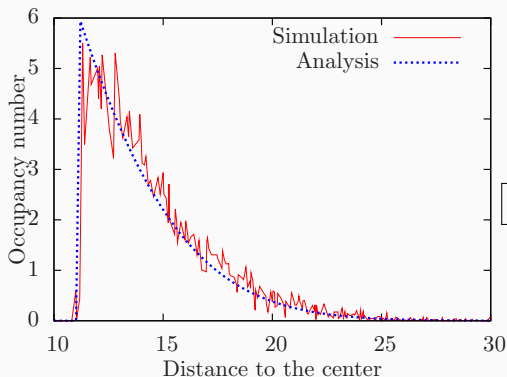
Whole area permitted

Center preferred

$k = 9, K = 10$

## COMPARISON BETWEEN ANALYTICAL AND SIMULATION RESULTS (2)

Agents with WTP under the threshold



$$u_k^*(X) = u_k^*(X) = \frac{\gamma}{K\alpha} \frac{A^0(X)}{\sum_{X' \in \Omega_k} A^0(X')}$$

Restricted area, center excluded

$$\Omega_k \equiv \{X \in \Omega \mid D(X) \geq d_c(k)\}$$

$k = 2, K = 10$



## Generalization :

- The monocentric city case can be refined
- The attractiveness linked to the presence of amenities can be redefined at the level of the neighborhood instead of the whole city level
  - ⇒ Limited space for each neighborhood, on which the  $k$ - agents can afford a good
  - ⇒ Local WTP thresholds

# HOUSING MARKET MODEL : ADAPTA- TION TO THE CITY OF PARIS

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## Housing market : Designing the model

### Housing market model : Adaptation to the city of Paris

## B.I.E.N DATA BASE

B.I.E.N. database, organized by the "Chambre des Notaires de Paris" : registers **real estate transactions for Paris and Ile De France** (flats, private houses, parking lots, fields, commercial or industrial buildings, ...).  
In 2000, 90% of the parisian transactions were registered.

For each transaction :

- the **location of the asset** (city, quarter, arrondissement in Paris, street and number and geocode)
- **intrinsic characteristics** (size, number of rooms, bathrooms,... age of construction, number of floors in case of a building and the amenities as lift...)
- **socio-economic profile** of the seller and the buyer (status, socio professional categories, geographic origins, age).

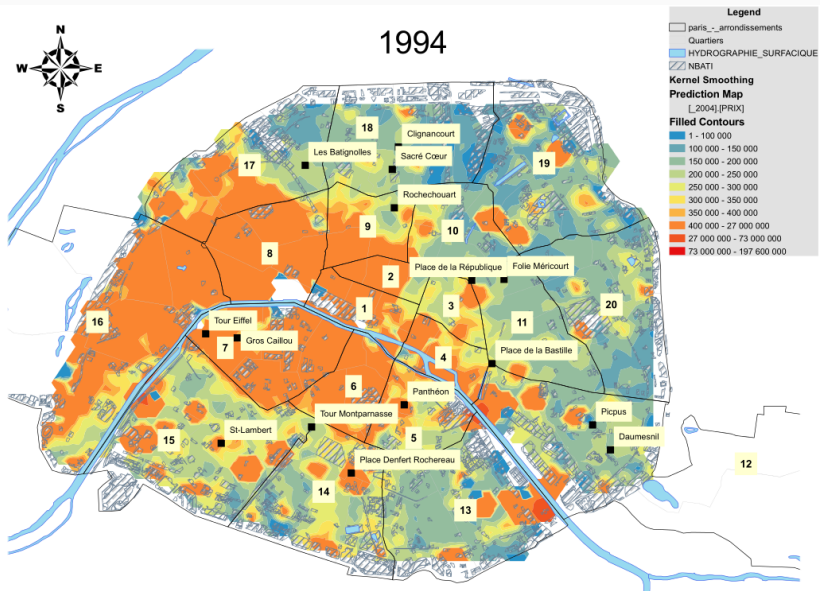
## DATA DESCRIPTION

- The data base contains information from 1990 until 2004
- We restricted the analysis to 1994 but it can be extended
- The number of registered transactions in 1994 is large enough
- This is an important year because it is situated before the fall in prices which affected Paris from 1995 to 2000.

## DATA DESCRIPTION

- During the year 1994, about 13 000 transactions were recorded in Paris
- The average price of a flat was 143 300 euros
- The standard deviation was around 90 000 euros
- Since the database does not contain the incomes of the buyers, we use the transaction prices as a proxy for income distributions (Friggit)

# PRELIMINARY STUDY : AVERAGED TRANSACTION PRICES



## COMPARISON BETWEEN THE SIMPLE MODEL AND THE DATA

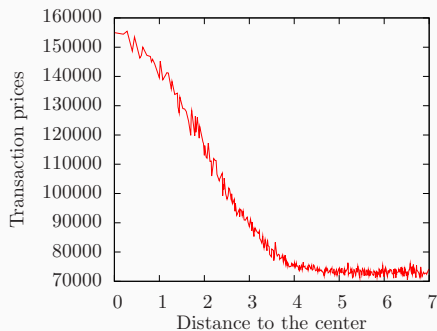
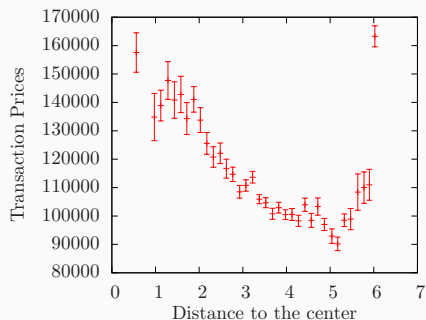
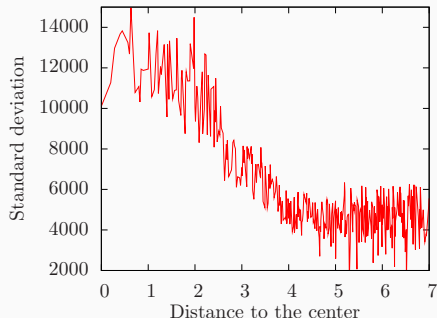
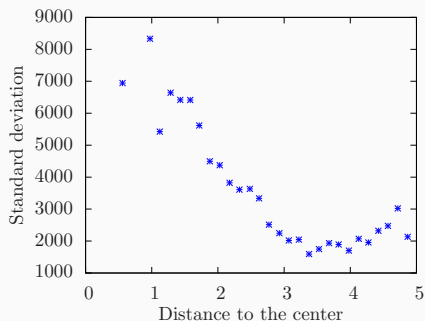


FIGURE 1 : Averaged transaction prices with respect to the distance to the center (left : data, right : model).



## COMPARISON BETWEEN THE SIMPLE MODEL AND THE DATA

The standard deviation of the transaction prices are plotted on Fig.2.



**FIGURE 2 :** Standard deviation of the averaged transaction prices with the distance to the center (left : data, right : model).

## GENERAL TRENDS

- A first obvious fact is that prices are high in the center, with a marked trend of decreasing prices as one moves away from the center
  - ⇒ The monocentric model could be seen as a first-order approximation of the Paris market
- There are obvious deviations from this general trend : prices are higher on the left bank than on the right
- In addition, prices are high in the 16th arrondissement on the very outskirts of Paris, creating a “hot-spot” of high prices far from the center
  - ⇒ The monocentric model must be corrected to take this into account

## ADAPTATION OF THE MODEL TO PARIS

- Adaptation of the **intrinsic attractiveness** : we develop the model to combine a general preference for the center together with preferences for some local particularities.
- **Adaption of the spatial representation** : we use a stylized map of Paris : three concentric zones of radius  $R_1, R_2, R_3$
- Each zone respectively has 4, 7 and 9 areas representing the arrondissements
- The ratios of the sizes of the simulated areas fit the real ratios

## ADAPTATION OF THE MODEL TO PARIS

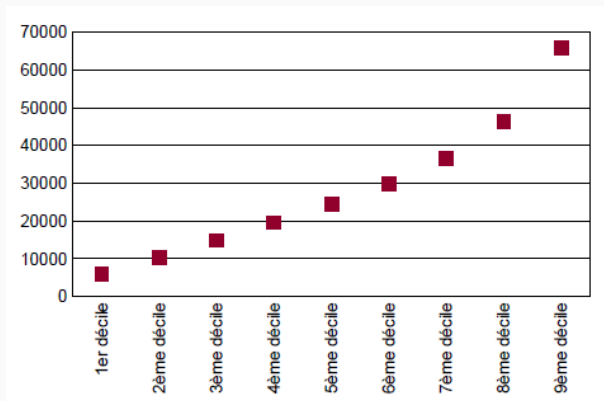


FIGURE 3 : Income distribution per house 2008 (INSEE)

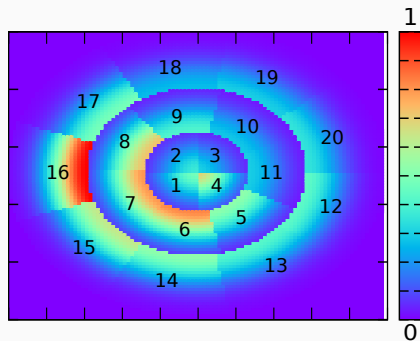
- Strong attractiveness  $\equiv$  more expensive arrondissements
- Expensive neighborhood  $\equiv$  to a strong demand  $\Rightarrow$  great attractiveness  
 $\Rightarrow$  Ratio of the squared mean transaction prices of the arrondissement to the squared maximal mean transaction price

$$A^{0,a}(X) = \left(\frac{P_{ref}^a}{P_{ref}^{max}}\right)^2 \exp\left(-\left(\frac{R^a - D(X)}{R}\right)\right)^2 \quad (10)$$

- $D(X)$  is the distance to the center
- $R^a \equiv$  shortest distance of the arrondissement to the center
- $P_{ref}^a$  mean transaction price of the arrondissement  $a$
- Some arrondissements expensive even far from the city center
- $R \equiv$  same order of magnitude as the distances to the center

# DISTRIBUTION OF ATTRACTIVENESS OVER THE CITY

Map of the intrinsic attractiveness



- Adjustment of the local intrinsic attractiveness (calibrated on the real estate transactions)

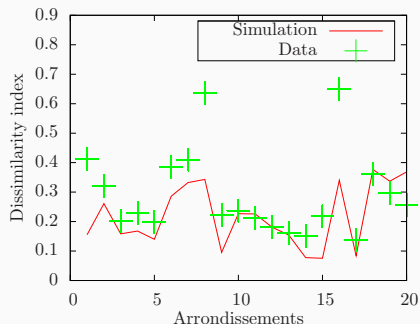
1. The above form and parametrization of the attractiveness give a stronger attractiveness to the more expensive arrondissements, even if they are far from the center of the city
2. The chosen calibration only provides the overall price trend of the arrondissements—it does not determine how the prices vary within a given arrondissement.

- Data published by the INSEE (French National Institute of Statistics and Economic Studies)
  - ⇒ Rate of moves of about 3,5%
- 375000 dwelling owners in Paris (main homes)
- About 13000 transactions in one year ⇒  $K = 13$  different WTP with

$$\frac{\gamma}{K} = \frac{1000}{L^2}, \alpha = 0.035 \text{ and } \epsilon = 0.18L^2$$



## Dissimilarity index computed for each arrondissement



## Proxy Income-transaction prices

- Weakest social mix observed in the 8, 16, 18, 19, 20-th arrondissements in the simulations and in the data.
- The arrondissements 8 and 16  $\equiv$  expensive arrondissements in Paris
- 18, 19, 20th  $\equiv$  more affordable places

## OUTCOMES AND OPENINGS

**Tractable** model → theoretical analysis achievable

**General** model → several parameters (for example, attractiveness) can be easily modified

⇒ **Framework** to study the housing market

- The first test of the model exhibits *segregative behavior* between different categories of agents and *social mix* in some cases :
- Emergence of hotspots via instabilities (gentrification)
- Polycentrism
- Long term dynamics can also be studied

## Relationship income distribution - vote distribution

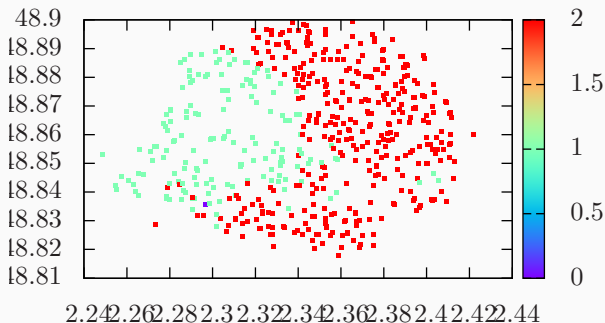


FIGURE 4 : Results of the regional elections : right vs left

Green  $\equiv$  Right

Red  $\equiv$  Left